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Self-avoiding walks on the lattices with a long-range-correlated disorder

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Abstract. We consider self-avoiding walks (SAW) on the diluted lattices on which the correlation function of the probability of occupied bonds (sites) obeys a power law r^{-a} for large separation r. The SAW is studied by a renormalisation-group expansion in $\varepsilon = 4-d$ and $\delta = 4-a$. We find the extended Harris criterion, for $n=0$ n-vector model or SAW, is that the disorder is irrelevant if $a\nu - 2 > 0$, which does not depend on the relation between a and d. If the disorder is relevant, the SAW has a new fixed point which has a correlation-length exponent $\nu = 2/a$.

1. Introduction

Recently there has been much interest in self-avoiding walks (SAW) on random lattices (Chakrabarti and Kertesz 1981, Kremer 1981, Derrida 1982, Harris 1983, Kim 1983, 1987, Rammal *et al* 1984, Roy and Chakrabarti 1987). These studies are concerned with SAW on lattices with randomness of short-ranged correlations. A common trend in studying physical phenomena (including SAW) on the random medium is to take the medium as a simple lattice with only short-range-correlated quenched randomness as in the ordinary percolation model (Essam 1980). However, such models fail to describe many important aspects of physical phenomena on the disordered medium (Trugman and Weinrib 1985, Trugman 1986) and several authors have tried to modify the simple lattice models in order to resolve this shortcoming.

Among variants of the disordered lattice models, we consider a model with a randomness of power-law correlation (Weinrib and Halperin 1983) to investigate the physical effect of a disorder which does not have short-ranged correlations. Let us make it clear what the power-law correlation means. Consider a bond (site) percolation problem, described by a variable θ_b at each bond (site) b. Then θ_b is 1 if the bond is occupied and 0 if the bond is vacant. Then the probability that a bond is occupied is $p = \langle\theta_b\rangle$, where $\langle\ \rangle$ means the configurational average. We define a correlation function $g(r_b, r_{b'})$ as $g(r_b, r_{b'}) = \langle\theta_b\theta_{b'}\rangle - \langle\theta_b\rangle^2$. By a power-law correlation we mean that the correlation function $g(r_b, r_{b'})$ falls off with distance as a power law as $g(r_b, r_{b'}) \sim |r_b - r_{b'}|^{-a}$.

In this paper we are considering SAW on the lattice with randomness of the power-law correlation. In the weakly dilute limit, where $p \to 1$ or far above the percolation threshold, we have shown that the critical properties of SAW on the lattice with randomness of only short-ranged correlations is in the same universality class as that on the non-random lattice (Harris 1983, Kim 1983). This may imply that a direct

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average over the SAW distribution function with the disorder of short-ranged correlations (Roy and Chakrabarti 1987) does not change the critical property of SAW, even though the logarithmic average over the SAW distribution function (Derrida 1982) does change it. But in reality the randomness could have various dispersions in characteristic length scales and various functional dependences. The main concern of this paper is thus to see whether or not the long-range-correlated randomness is a relevant perturbation for the critical properties of SAW (in the sense of renormalisation group (RG)).

Our treatment of this problem is based on an RG(ε, δ) expansion which has been used by Weinrib and Halperin (1983) to study the critical phenomena of the n-vector model with long-range-correlated quenched disorder in case of $n \geqslant 1$. Here $\varepsilon = 4-d$, $\delta = 4-a$ and d is the spatial dimension. Our main concern is the $n=0$ case of the n-vector model, because the generating function for SAW corresponds to the partition function of an n-vector model of magnetism in the limit $n \to 0$ (de Gennes 1979). In the $n \geqslant 1$ case, the extended Harris criterion (Weinrib and Halperin 1983) for the long-range-correlated disorder is that for $a<d$ the disorder is irrelevant if $a\nu - 2 > 0$, while if $a>d$ the usual Harris criterion (Harris 1974) holds. In contrast when $p \to 1$ the usual Harris criterion does not hold for the $n=0$ n-vector model (Harris 1983, Kim 1983) and thus we need a new extended Harris criterion for the $n=0$ n-vector model. As we shall see the extended Harris criterion for the $n=0$ case is that the disorder is irrelevant if $a\nu - 2 > 0$ regardless of the relation between a and d.

2. Field theory for the $n=0$ n-vector model

The lattice randomness of the power-law correlation is equivalent to the random variation of the local transition temperature $T_c(x)$ of the n-vector model as discussed by Weinrib and Halperin (1983) who have shown that the effective Hamiltonian of an n-vector model with random variation of the local transition temperature with the power-law correlations is

$$H_{\text{eff}} = H_0(r, u) + H_r \tag{1}$$

where

$$H_0(r, u) = \int \mathrm{d}^d x \sum_{\alpha=1}^{m} \left(\tfrac{1}{2} r|\boldsymbol{\phi}^\alpha(x)|^2 + \tfrac{1}{2}|\nabla \boldsymbol{\phi}^\alpha(x)|^2 + u|\boldsymbol{\phi}^\alpha(x)\cdot\boldsymbol{\phi}^\alpha(x)|^2\right) \tag{2}$$

and

$$H_r = -\int \mathrm{d}^d x\, \mathrm{d}^d y\, g(x-y) \sum_{\alpha,\beta} |\boldsymbol{\phi}^\alpha(x)|^2 |\boldsymbol{\phi}^\beta(y)|^2 \tag{3}$$

where $\boldsymbol{\phi}^\alpha(x) = (\phi_1^\alpha(x), \phi_2^\alpha(x), \ldots, \phi_n^\alpha(x))$, α and β are replica indices which assume the values $1, 2, \ldots, m$, and the limit $m \to 0$ is implied. $g(x-y) = g(r)$ satisfies the long-range dependence r^{-a}. $H_0(r, u)$ is the Hamiltonian for the non-random n-vector model because there is no replica-mixing term. H_r, which is a replica-mixing term, represents the effect of randomness (Weinrib and Halperin 1983). Upon taking Fourier transforms $\boldsymbol{\phi}^\alpha(x) \to \boldsymbol{\phi}^\alpha(q)$, the replica-mixing interaction H_r is rewritten as

$$H_r = -\int_q \int_{q'} \int_k \sum_{\alpha,\beta} g(k) \boldsymbol{\phi}^\alpha(q+k)\cdot\boldsymbol{\phi}^\alpha(-q)\boldsymbol{\phi}^\beta(q-k)\cdot\boldsymbol{\phi}^\beta(-q') \tag{4}$$

where $\int_q$ means the usual shorthand notation for $\int \mathrm{d}^d q/(2\pi)^d$ and $g(k)$ is the Fourier transform of $g(r)$. With $g(r) \sim r^{-a}$ for large r (Weinrib and Halperin 1983)

$$g(k) \sim v + wk^{(a-d)} \tag{5}$$

when $k \to 0$. In H_r there are two quartic interactions. The quartic interaction proportional to v is the effect of randomness of the short-ranged correlation and it is not a relevant perturbation for the $n=0$ n-vector model, because if we take both the SAW limit ($n \to 0$) and the replica limit ($m \to 0$) simultaneously the quartic perturbation v is the same as the interaction u in H_0 (Kim 1983). In the limits $n \to 0$ and $m \to 0$ the effective Hamiltonian is equivalent to

$$H_{\mathrm{eff}} = H_0(r, \bar{u}) + H_r(w) \tag{6}$$

and

$$H_r(w) = -w \int_q \int_{q'} \int_k \sum_{\alpha,\beta} k^{(a-d)} \boldsymbol{\phi}^\alpha(q+k) \cdot \boldsymbol{\phi}^\alpha(-q) \boldsymbol{\phi}^\beta(q-k) \cdot \boldsymbol{\phi}^\beta(-q') \tag{7}$$

where $\bar{u} = u - v$. The only relevant interaction in our case for the randomness is the w interaction which represents the randomness of long-ranged correlation. Before we begin our RG study of Hamiltonian (6), the extended Harris criterion for $n=0$ can be inferred by heuristic arguments (Harris 1974, Weinrib and Halperin 1983). If the w interaction in (6) is regarded as a perturbation to the non-random Hamiltonian $H_0(r, \bar{u})$, the free energy corresponding to H_{eff} of (6) can be expanded around the non-random free energy. Averaging over quenched random configurations, we obtain near the critical point

$$\langle F \rangle_0 = F_0 + \langle H_r(w) \rangle_0 \tag{8}$$

where $\langle - \rangle_0$ means the average over non-random theory and F_0 is the free energy without randomness. The usual scaling estimates for F_0 and $\langle H_r(w) \rangle_0$ (Amit 1984, Harris 1983) near the critical point are

$$F_0 \sim t^{2-\alpha} \qquad t \equiv (T - T_c)/T \tag{9a}$$

and

$$\langle H_r(w) \rangle_0 \sim w \int_k t^{-\alpha} f(k\xi) k^{(a-d)} \sim w t^{-\alpha + a\nu}. \tag{9b}$$

The randomness of long-ranged correlations for the $n=0$ n-vector model is relevant when $\langle H_r(w) \rangle_0 \gg F_0$, i.e. when $a\nu - 2 < 0$. In contrast for the model $n \geqslant 1$ we must consider $H_{\mathrm{eff}}(r, u, v, w)$ (see (1) and (5)) instead of H_{eff} of (6) and the extended criterion must be dependent on the relation between a and d as is stated in § 1 (Weinrib and Halperin 1983).

3. RG recursion relations in ε and δ

Proceeding in the usual way as shown by Weinrib and Halperin (1983), the differential recursion relations of parameters r, $\bar{u}$ and w for the Hamiltonian (6) to one-loop order (Amit 1984), i.e. order of $\mathrm{O}(\varepsilon, \delta)$ with $\delta = \mathrm{O}(\varepsilon)$, are as follows:

$$\mathrm{d}r/\mathrm{d}l = 2r + 8(\bar{u} - w) \tag{10a}$$

$$d\bar{u}/\mathrm{d}l = \varepsilon\bar{u} - 32\bar{u}^2 + 48\bar{u}w - 16w^2 \tag{10b}$$

$$dw/\mathrm{d}l = \delta w - 16\bar{u}w + 16w^2. \tag{10c}$$

The recursion relations in (10*a*)-(10*c*) can be derived from the recursion relations (Weinrib and Halperin 1983) for the model $n \geqslant 1$ in which the RG parameters are r, u, v and w by taking limits $n \to 0$ and putting $\bar{u} = u - v$ (Kim 1983). The physical region of parameter space is $\bar{u} > 0$ and $w > 0$ (Weinrib and Halperin 1983, Kim 1983). The fixed points of the recursion relations in (10) are presented in table 1 and the eigenvalues of those fixed points in table 2. When $d < 4$ or $\varepsilon > 0$ the Heisenberg fixed point is stable if $\varepsilon > 2\delta$. To the order of ε and δ this becomes $a\nu_{\mathrm{H}} - 2 > 0$, where $\nu_{\mathrm{H}} = \frac{1}{2}(1 + \varepsilon/8)$ is the correlation exponent of the Heisenberg fixed point (II). When $d > 4$ or $\varepsilon < 0$ the Gaussian fixed point is stable if $\delta < 0$. To the order of ε and δ this also becomes $a\nu_{\mathrm{G}} - 2 > 0$, where $\nu_{\mathrm{G}} = \frac{1}{2}$ is the correlation exponent of the Gaussian fixed point. This shows that the non-random critical behaviour crosses over to the long-range-disorder critical behaviour when $a\nu - 2 < 0$ regardless of the relation between a and d, which we have derived heuristically in § 2. When $d < 4$ the long-range-disorder fixed point is stable when $\varepsilon > \delta > \varepsilon/2$ and is in the physical region, because $\bar{u}^* > 0$ and $w^* > 0$. The correlation-length exponent ν in this case is $\nu_{\mathrm{L}} = 2/a$.

When $\delta \to \varepsilon$ or $a \to d$ the $\bar{u}^*$ and w^* of the long-range-disorder fixed point diverge and the perturbative expansion of the recursion relations may break down. The recursion relations to one-loop order have a degeneracy in that $\bar{u} - w$ appears in both expressions in (10*b*) and (10*c*). A similar degeneracy also appears to one-loop order in that $v + w$ appears in recursion relations for the $n \geqslant 1$ n-vector model (Weinrib and Halperin 1983). This leads the apparent divergence of $\bar{u}$ and w of the long-range-disorder fixed point in table 1. A similar divergence also occurs at both the long-range unphysical fixed point and the long-range-disorder fixed point in the $n \geqslant 1$ case (Weinrib and Halperin 1983). To two-loop order this degeneracy may be resolved if this case is similar to the short-range-disorder fixed point for the Ising model ($n = 1$) in which the degeneracy in u and v to one-loop order has been resolved to two-loop order (Jayaprakash and Katz 1977). If this case is not similar to the degeneracy of the random fixed point for the Ising model, then the degeneracy might not be solved by

Table 1. Gaussian, Heisenberg and long-range-disorder fixed points of the recursion relations in (10).

	Fixed points		
	Gaussian	Heisenberg	Long-range-disorder
r^*	0	$-\varepsilon/8$	$-\delta/4$
$\bar{u}^*$	0	$\varepsilon/32$	$\delta^2/16(\varepsilon-\delta)$
w^*	0	0	$\delta(2\delta-\varepsilon)/16(\varepsilon-\delta)$

Table 2. Eigenvalues of the fixed points of the recursion relations in (10).

	Eigenvalues		
	Gaussian	Heisenberg	Long-range-disorder
$\lambda_r = 1/\nu$	2	$2-\varepsilon/4$	$2-\delta/2$
λ_1	ε	$-\varepsilon$	$\frac{1}{2}\{(\varepsilon-4\delta)$
λ_2	δ	$\delta-\varepsilon/2$	$\pm[8(\delta-\varepsilon/4)^2+\varepsilon^2/2]^{1/2}\}$

our approach. For both $n \geq 1$ and $n = 0$ cases this degeneracy is an open question. When $d > 4$ and $\varepsilon < 0$ a similar problem occurs, but we cannot solve this problem within our perturbative approach.

According to a recent classification by Roy and Chakrabarti (1987), this paper discusses a direct average over the SAW distribution function where there is a randomness of long-ranged correlations. The logarithmic average over the SAW distribution function (Derrida 1982) is a more difficult question because even short-range-correlated disorder does have some relevancy.

Throughout this paper we have treated the weakly dilute case in which $p \to 1$. Near the percolation threshold the crossover behaviour of SAW is quite complex (Kremer 1981, Kim 1987) and we cannot apply our extended criterion to SAW around percolation threshold.

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